Robot Wrist Configurations, Mechanisms and Kinematics

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Abstract: This paper discusses a general method for determining the kinematic performance of spherical robot wrists. Different wrist types considered include R-P-R (roll-pitch-roll) and P-Y-R (pitch-yaw-roll) wrists. Singularity conditions are indicated for both cases. For R-P-R wrists, singularity is defined by the ratio of the angular velocity of each motor to the velocity around the degeneracy axis. For P-Y-R wrists, singularity is identified both by analyzing the Jacobian matrix and by analyzing the relative velocities of the wrist components.

1 Introduction

Robot wrists are designed to provide orientation to the end effector. They should preferably be centered around a point, that means to constitute "spherical joints", because the resulting configuration is more dexterous and less cumbersome than the other configurations.
Robot wrists should have a low "degeneracy level", that represents the region where some rotations around certain fixed axes in the cartesian space are forbidden, or require very high speeds of the actuators. Degeneracy occurs when the three axes of the spherical wrist joints are contained in one plane. The degeneracy condition implies that it is not possible to follow the shortest path in producing certain orientations according to a specified sequence in the work envelope without exceeding the maximum possible velocity for certain joints [Paul and Stevenson (1983)], [Huang and Milenkovic (1987)], [Treveljian et al. (1986)].
They are two main principles concerning the make of wrists. One is the R-P-R (roll-pitch-roll) method; the other is the P-Y-R (pitch-yaw-roll) method [Rivin (1988)]. R-P-R wrists are based on the application, through conical gears, of the Euler formulas for successive rotations of bodies with respect to the others [Litvin and Zhang (1986)], [Romiti and Sorli (1992)]. P-Y-R wrists act in different way. Two axes (P and Y) are fixed in space (with respect to the last robot arm): the adjiustment of rotations to comply to kinematic constraints is obtained by free rotations of bodies one with respect to the others [Milenkovic (1987)], [Rosheim (1989)], [Romiti and Raparelli (1993)].

2 R-P-R wrists

R-P-R wrists consist of three arms connected in sequence via three rotary joints. The kinematic connection of each arm with the preceding arm can be described using the transfer matrices ${}^{i-1}A_i$; if α_{i-1} is the angle which brings joint axis z_{i-1} to z_i through a rotation around axis x_{i-1}, and if θ_i is the angle which brings axis x_{i-1} to x_i through a rotation around axis z_i we have:

$$ {}^{i-1}A_i = \mathrm{Rot}(x_{i-1}, \alpha_{i-1})\mathrm{Rot}(z_i, \theta_i) = \begin{vmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} \end{vmatrix} \tag{1}$$

Angular velocities of the successive arms are given by the expression:

$$\vec{\omega}_i = \vec{\omega}_{i-1} + \vec{\omega}_{(i-1,i)} \tag{2}$$

where $\vec{\omega}_{(i-1,i)}$ describes the angular velocity of body *i* relative to body *i-1*. Assuming that the three sets are arranged so that the three degrees of freedom θ_i occur around axes z_i, we have:

$$\vec{\omega} = \dot{\theta}_1\vec{k}_1 + \dot{\theta}_2\vec{k}_2 + \dot{\theta}_3\vec{k}_3 \tag{3}$$

Remembering (1), we then have (where $C_i=\cos(\theta_i)$ and $S_i=\sin(\theta_i)$ with i=1,2):

$$\vec{\omega} = \begin{vmatrix} \omega_x \\ \omega_y \\ \omega_z \end{vmatrix} = \begin{vmatrix} 0 & -S_1 & C_1S_2 \\ 0 & C_1 & S_1S_2 \\ 1 & 0 & C_2 \end{vmatrix} \begin{vmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{vmatrix} \tag{4}$$

P. Kopacek (ed.), *Robotics in Alpe-Adria Region*

We will examine a spherical wrist developed at our laboratory. It has three axes which intersect at right angles (Figure 1a). Roll movement of arm 1 is controlled by the motor with angular velocity $\dot{\phi}_1$. Arm 2 is rotated around axis z_2 by the bevel gear set consisting of gears N_1 (connected to the motor with angular velocity $\dot{\phi}_2$) and N_2 (connected to arm 2). The roll movement takes place around axis Z_3 and is produced via bevel gears N_3, N_4, N_5 and N_6.
Indicating the axis perpendicular to z_2 and z_3 with x' and using the notation shown in figure 1b which defines the configuration assumed by the three sets of cartesian axes, we can write:

$$\begin{aligned} \omega_{x'} &= C_1\omega_x + S_1\omega_y \\ \omega_{y'} &= C_1\omega_y - S_1\omega_x \\ \omega_{z'} &= \omega_z \end{aligned} \quad (5)$$

As the wrist has a single center, we have:

$$\frac{N_2}{N_1} = \frac{N_4}{N_3} = \frac{N_5}{N_6} \quad (6)$$

$$\begin{aligned} \dot{\phi}_2 - \dot{\phi}_1 &= \frac{N_2}{N_1}\dot{\theta}_2 \\ \dot{\phi}_3 - \dot{\phi}_1 &= \frac{N_4}{N_3}\dot{\theta}_4 \\ \dot{\phi}_1 &= \dot{\theta}_1 \end{aligned} \quad (7)$$

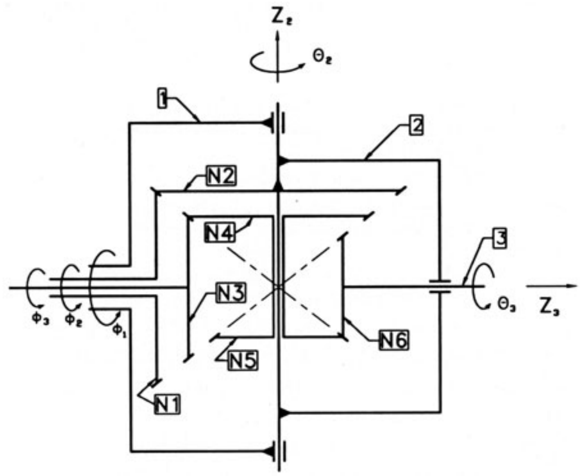

Fig. 1a. Kinematic scheme of a R-P-R wrist

The common angular speed of wheels N_4 and N_5 around axis z_2 is $\dot{\theta}_4$. The angular speed of wheel N_3, $\dot{\theta}_3$, is then given by:

$$\dot{\theta}_4 - \dot{\theta}_2 = \frac{N_6}{N_5}\dot{\theta}_3 \quad (8)$$

The relations between velocities $\dot{\theta}_i$ and $\dot{\phi}_i$ with i = 1,2,3 are thus:

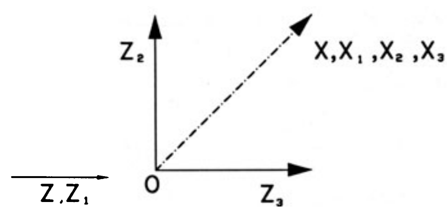

Fig. 1b. Wrist reference systems

$$\begin{aligned} \dot{\theta}_1 &= \dot{\phi}_1 \\ \dot{\theta}_2 &= \frac{N_1}{N_2}(\dot{\phi}_2 - \dot{\phi}_1) \\ \dot{\theta}_3 &= \frac{N_5}{N_6}\left[\frac{N_3}{N_4}(\dot{\phi}_3 - \dot{\phi}_1) - \frac{N_1}{N_2}(\dot{\phi}_2 - \dot{\phi}_1)\right] \end{aligned} \quad (9)$$

Introducing equations (5) in (7) gives:

$$\begin{aligned} \dot{\phi}_1 &= \omega_z - \frac{C_2}{S_2}\omega_{x'} \\ \dot{\phi}_2 &= \omega_z - \frac{C_2}{S_2}\omega_{x'} + \frac{N_2}{N_1}\omega_{y'} \\ \dot{\phi}_3 &= \omega_z + \frac{1}{S_2}\left(\frac{N_4N_6}{N_3N_5} - C_2\right)\omega_{x'} + \frac{N_4}{N_3}\omega_{y'} \end{aligned} \quad (10)$$

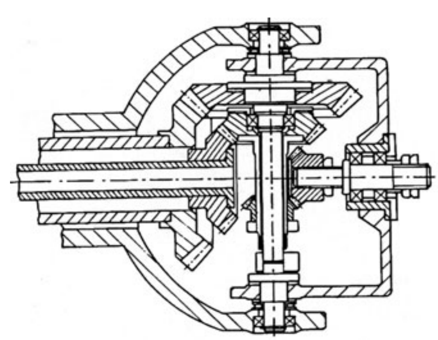

Fig.2. R-P-R wrist constructive example